I.S.S.N: 1885-6888

ECONOMIC ANALYSIS WORKING PAPER SERIES

Rate Equations Approach to Simulate World Population Trends

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Working Paper 7/2011

UAM UNIVERSIDAD AUTONOMA DE MADRID

DEPARTAMENTO DE ANÁLISIS ECONÓMICO: TEORÍA ECONÓMICA E HISTORIA ECONÓMICA

Rate Equations approach to simulate World population trends

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Abstract: According to UN statistical data and projections world population will begin to decrease by the middle of this century. This paper uses rate equations (fully analogous to those employed in condensed matter physics) to simulate the time evolution of world population, making use of UN population data in the time interval 1900-2010, and to extrapolate the evolution of world population into the near future. This approach has not been used in economics and population dynamics. The simulation predicts a population decline by mid-century. The economic consequences of population decline would be far reaching.

Keywords: rate equations, demographic transition; population trend simulation; world population decline

JEL: C65, J11

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1. Introduction

The UN data on world population, fertility rates, birth and death rates provide abundant and reliable information to investigate present population trends, and to make guesses about the future. These data can be complemented with vital statistical data from the United Nations Department of Economic and Social Affairs.[1]

In what follows we introduce population rate equations (Duan et al. 2010) appropriate to describe the evolution of the population of a two level system in terms of the known birth rate $\left(BR = r_b\right)$ and death rate $\left(DR = r_d\right)$, under the assumption that total biomass is approximately conserved. Global biomass would play the role of a universal constant that somehow provides an upper limit to population, at least in the short-medium term. Obviously global biomass can change within certain limits depending on environmental conditions but it is not completely elastic. In this paper we use a rate equations approach to describe world population dynamics which involve a conservative principle. Within a human generation (about 25 years) the world total biomass may be expected to change at most a a factor of 2 to 3 –except if a nuclear war, global pestilence or the collision of a comet with our planet take palce.

The Earth has a biosphere capable of sustaining life: plants, animals and men. It has been estimated that the total amount of living matter on Earth is about 1.4×10^{15} Kg according to the World Atlas of Biodiversity (Groombridge and Jenkins, 2002).[2] Men, like other higher animals, live on a diet of carbohydrates, fats and proteins. As it is well known, in the last instance, the chain of life in our planet begins with photosynthesis by the green plants. According to FAO (see Vian Ortuño 1994) the distribution of vegetal biomass is roughly the following: 46% forests and lower wild vegetation; 10% pastures and steppes; 1.8% deserts; 6.2% crops; 2.9% biomass in continental waters; 33% biomass in the oceans. At present the ratio of human to total biomass is roughly

[1] Population Division. http://www.un.org/esa/population/unpop.htm

[2] Asimov 1972, p. 795, give a figure of 2×10^{16} Kg.

$$\frac{\text{Human Biomass}}{\text{Total Biomass}} \approx \frac{60 \times 6.8 \times 10^9}{1.4 \times 10^{15}} \approx 2.9 \times 10^{-4}$$

The consumption (and large-scale waste) of energy is of course an important factor affecting world population. However, it should be kept in mind that converting directly 1/6000 of the radiant energy coming from the Sun would provide sufficient electrical or chemical energy to satisfy present world energy needs. Fusion energy research may someday contribute effectively to provide abundant, safe and clean energy.

Under the above mentioned assumption, the solutions of rate equations are shown to simulate quantitatively well the actual time evolution of the world population $P(t)$ in the second part of the twentieth century, and to provide reasonable grounds for estimating quantitatively the short term evolution of world population.

The paper is organized as follows: in section 2 we derive the rate equations that we use to describe and predict world population in the 21st Century; section 3 offers a brief analysis of World population data from 1950 to 2010; section 4 presents the results of applying rate equations to describe quanlitatively world population trends; and finally, section 5 is the conclusion.

2. Rate Equations

We consider a certain step change (up or down) in population associated with a correlative change (up or down) in human biomass $\Delta M(Kg)$ such that the final change in population (after the step is over) is given by

$$\Delta N \cong \Delta M\ (Kg)/60\ (Kg) = N_2(t) + N_1(t) \tag{1a}$$

Where $N_2(t)$ is the number of individuals (men and women) actually alive at time *t*, and $N_1(t)$ the number of individuals potentially alive at the same time. $N_1(t)$ corresponds, therefore, to the amount of biomass potentially convertible into human mass at that time. Time *t* is in the interval (beginning) $t_i \le t \le t_f$ (end),where t_f is sufficiently larger than the relevant characteristic time τ^* for the step (up or down). We can assume ΔN to be in the range $10^9 < N < 10^{11}$, in principle, and N_{RL} (the bakground replacement level population at $t < t_i$) to be of the same order of magnitude.

During the transition between $N(0) = N_{RL}$ (original replacement level) to $N(t_f) = N_{RL} + \Delta N(t_f)$ (final replacement level) the excess human biomass is distributed between the two levels in such a way that always

$$N_2(t) + N_1(t) = \Delta N(t_f) \tag{1b}$$

At any given time there is a certain birth rate, $BR = r_b$, governing the transition of human biomass from level 1 to level 2, and a certain death rate, like wise governing the transition from level 2 to level 1.

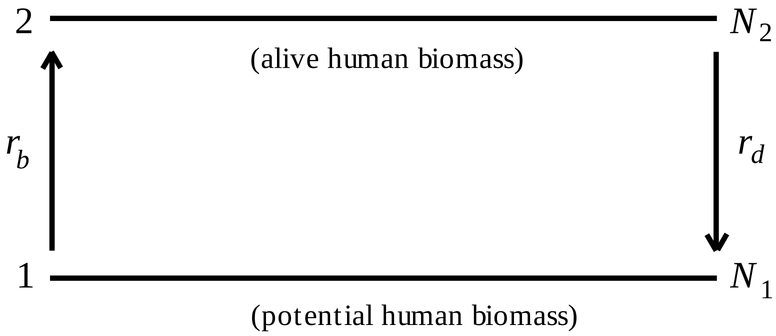

Figure 1: The world as two level system to describe step (+/-) over a certain population background (Replacement Level).

We can define the respective transition probabilities (say per 100 or 1000 persons) per unit time as

$$p_{12} = r_b = r_0 e^{\alpha} \text{ (birth rate)} \tag{2}$$

$$p_{21} = r_d = r_0 e^{-\alpha} \text{ (death rate)} \tag{3}$$

where $r_0 \equiv p_0 e^{-\alpha_0}$ could be viewed as the product of an attempt frequency p_0 (inverse of a natural characteristic time 2τ) modified by a reducing factor $e^{-\alpha_0} \leq 1$.

Alfa (α) in equations (2) and (3) can be taken as a kind of 'growth potential', determinant of the increase (or decrease) of population in the time interval considered. Then,

$$r_b \times r_d = r_0^2 = (1/2\tau)^2 \text{, hence } 1/\tau = (r_b \times r_d)^{1/2} \tag{4}$$

$$r_b / r_d = e^{2\alpha} \text{ , hence } \alpha \equiv \frac{1}{2} \ln\left(r_b / r_d \right) \qquad (5)$$

The population rate equation can be written therefore as

$$\frac{dN_2}{dt} = N_1 p_{12} - N_2 p_{21} \qquad (6)$$

$$\frac{dN_1}{dt} = -N_1 p_{12} + N_2 p_{21} \qquad (7)$$

Subtracting Eq. (7) from Eq. (6) we get

$$\frac{d\left(N_2 - N_1\right)}{dt} = \left(N_2 + N_1\right)\left(p_{12} - p_{21}\right) - \left(N_2 - N_1\right)\left(p_{12} + p_{21}\right) \qquad (8)$$

wich, taking into account that $\left[N_2(t) - N_1(t) \right] = \Delta P(t)$ is the increse in live population at time *t*, and $\left[N_2 + N_1 \right] = \Delta N (\text{constant})$, can be rewritten as

$$\frac{d\Delta P}{dt} = N 2 r_0 \sinh\alpha - \Delta P 2 r_0 \cosh\alpha \qquad (9)$$

Using $p(t) = \Delta P(t) / N$, dimensionless, Eq. (9) becomes

$$\frac{dp(t)}{dt} = \frac{1}{\tau}\left[\sinh\alpha - p(t)\cosh\alpha \right] \qquad (10)$$

The general solution of this linear differential equation is

$$p(t) = \mathrm{e}^{-\int (\cosh\alpha/\tau) dt} \left[\frac{\sinh\alpha}{\tau} \mathrm{e}^{\int (\cosh\alpha/\tau) dt} dt + C \right] \qquad (11)$$

In particular, for a step (up or down) in growth potential, say from $\alpha = 0$ at $t = 0$ to $\alpha \neq 0$ at $t > 0$, the integrals in Eq. (11) are strightforward, and we get

$$p(t) = e^{-(\cosh\alpha/\tau) dt} \left[\tanh\alpha \times e^{(\cosh\alpha/\tau) dt} + C \right] \qquad (12)$$

which, from $p(0) = 0$ at $t = 0$ leads to

$$C = -\tanh\alpha \qquad (13)$$

resulting in

$$p(t) = \tanh\alpha\left[1 - e^{-(\cosh\alpha/\tau)dt}\right] \tag{14}$$

Therefore, using $p(t) \equiv \Delta P(t)/N$ we finally get

$$P(t) = P_{RL} + \Delta P_{max}\tanh\alpha\left[1 - e^{-(t-t_i)/\tau^*}\right] \tag{15}$$

where $\tau^* = \tau/\cosh\alpha$, for a step up ($\alpha > 0$) in population.

3. Analysis of World Population Data (1950-2010)

In order to analyze in detail the available UN world population data (1950-2050), and to make some qualitative considerations about future trends, it is convenient first to introduce an empirical relationship between the birth rate $(BR = r_b)$, defined as the number of births per year per 100 population, and the fertility rate (*FR*), defined as the average total number of children per woman.

Fig. 2 gives the data for birth rate vs fertility rate for China, India, USA and Russia (1995-2010) summarized in Table 1. The relationship

$$(BR) = 0.72(FR) \times 10^{-2} \tag{16}$$

is very approximately fulfilled. Here the proportionality factor between (*BR*) and (*FR*) has been slightly corrected to take into account that the global female to male population in those countries is about. (% female)/(% male) $\cong$ 0.935.

Table 1: Birth rate and fertility rate for various countries (1995-2010).

	China		India		USA		Russia	
Year	BR	FR	BR	FR	BR	FR	BR	FR
1995	1.82	2.3	2.93	3.9	1.57	2	1.07	1.6
2000	1.62	1.8	2.59	3.2	1.47	2	0.93	1.4
2005	1.3	1.7	2.33	2.9	1.41	2.1	0.96	1.3
2010	1.24	1.7	2.23	2.8	1.4	2.1	1.05	1.4

Source UN.

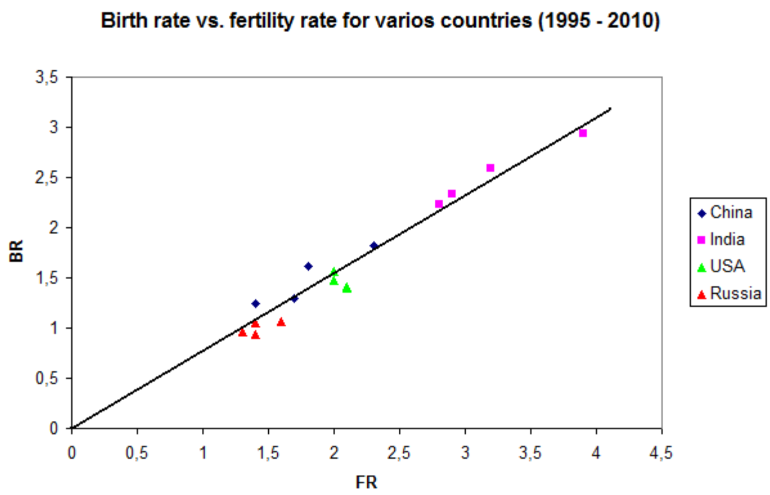

Figure 2: Birth rate per 100 population versus Fertility rate for China, India, USA and Russia (1995-2010). An excellent correlation is found by means of $(BR) = (0.72)(FR)\times 10^{-2}$. (Source: www.unpopulation.org)

Table 2: UN demographic data (1950-2010) and projections (2010-2040).

Year	*FR* (UN)	*GR* (UN)	*BR* (Eq.16)	*DR* (Eq.17)	x BR/DR	α Eq.5	τ^{-1} Eq.4	τ^{*} $\tau/\cosh\alpha$
1950	5.02	1.80	3.61	1.81	1.99	0.345	0.0512	18.42
1960	4.97	1.97	3.58	1.61	2.22	0.400	0.0480	19.28
1970	4.48	1.94	3.23	1.29	2.51	0.460	0.0407	22.17
1980	3.57	1.71	2.57	0.86	2.99	0.547	0.0297	29.15
1990	3.03	1.50	2.18	0.68	3.20	0.582	0.0244	34.93
2000	2.48	1.07	1.79	0.72	2.50	0.457	0.0226	39.98
2005	2.22	0.87	1.60	0.73	2.19	0.393	0.0216	42.98
2010	2.05	0.74	1.48	0.74	2.01	0.348	0.0208	45.21
2020	1.87	0.48	1.35	0.87	1.55	0.220	0.0216	45.19
2030	1.71	0.17	1.23	1.06	1.16	0.074	0.0229	43.62
2040	1.59	0.10	1.14	1.04	1.10	0.046	0.0219	45.67

UN data for world fertility rates (*FR*) and the world growth rates (*GR*) for 1950-2010 are availabe at United Nations web,[16] from which the birth rate ($BR = 0.72\times FR$) and the death rate ($DR = BR - GR$) are directly obtained. The corresponding numbers for growth potential $\alpha = \ln(BR/DR)/2$ and inverse characteristic time $1/\tau = 2(BR\times DR)^{1/2}\times 10^{-2}$ are given in subsequent columns. Projected UN rates for subsequent years are given for completeness. It can be seen that, in spite of the fact that (*BR*) and (*DR*) vary considerably with time between 1950 and 2000, the numerical values for $x = BR/DR \approx 2.57$ and $\alpha = \ln(BR/DR)/2 \approx 0.471$ can be used as representative values for the half century through which a large step up in population is taking place. The numerical value for τ, however,varies smoothly from about $\tau = 18.4$ years in 1950 to about $\tau = 40.18$ years in 2000. This can be correlated to the increase in life expectancy and possibly to a marked delay in the lifegiving age for women. (Table 2 gives in the first two columns the UN world fertility rates (*FR*) and the world growth rate (*GR*) for 1950-2010, from which the birth rate $BR = 0.72 \times FR$ and the death rate $DR = BR - GR$ are directly obtained.)

Using the data from the Population Division of the UN Department of Economic and Social Affairs we may analize in detail the *step up* in world population taking place between 1950 and 2010, and then, taking into account that x (and α) begin to decrease in the year 2000, we can guess on the time for the incipient *step down* in world population.

4. Results

In Fig. 3, the normalized population $p(t) = \tanh\alpha\left[1 - e^{t/\tau^*}\right]$, $\tau^* = \tau / \cosh\alpha$, is given as a function of t/τ^* for various values of $x = r_b / r_d$, ($\alpha = 1/2 \ln x$). It can be seen that a replacement level (*RL*) is achieved in all cases for $t/\tau^* > 3$, and that the population at the replacement level grows in proportion to $\tanh\alpha$.

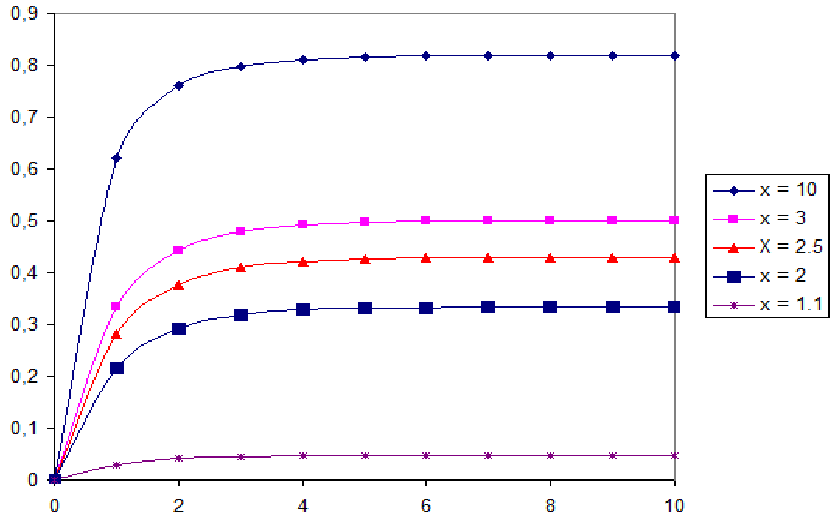

Figure 3: Normalized population increase versus normalized time for various $x = r_b / r_d$. Curves: $p(t) = \tanh\alpha\left[1 - \exp(-t/\tau^*\right]$; $\tau^* \approx \tau / \cosh\alpha$. (For different values of $x = r_b / r_d$, $\alpha = \ln x / 2$.)

Figure 4 gives the birth rate (*BR*) and the death rate (*DR*) per year per 100 population as a function of time: actual UN data (1950-2010), UN (2004) projections for the period 2010-2050 are also given. It is seen that, in the interval (1950-2000), *DR* decreases monotonously and somoothly up to 1990. In the interval 1990-2010 a change in *DR* is taking place, probably related to the fact that in some countries, like Japan, the transient surplus population connected with the sustined increase in life expectancy , accompanied by the decrease in effective fertility rate, is beginning to fade away. During this interval the world population still increases slowly but it is leveling out. There is no such a thing as an exponential increase.

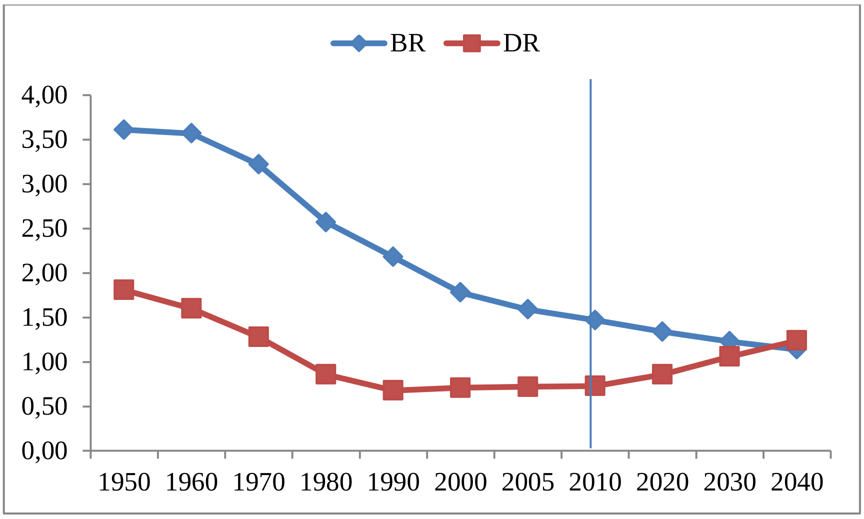

Figure 4: Birth rate ($\times 10^2$) and death rate ($\times 10^2$) UN actual data (1950-2010) and UN projections (2010-2050). See Table 2.

Figure 5 displays the *population growth potential* α (dimensionless) and the *characteristic time* τ (years) in the same time span. α remains practically constant up to $t = 2000$, and then begins to drop up to 2010. A linear extrapolation of the actual data for $\alpha(t)$ up to 2020 suggests a population decrease at about mid-century. On the other hand τ^* evolves smoothly from $\tau^*(1950) \approx 18.4$ years to $\tau^*(2000) \approx 40.0$ years and then seems being to stabilize.

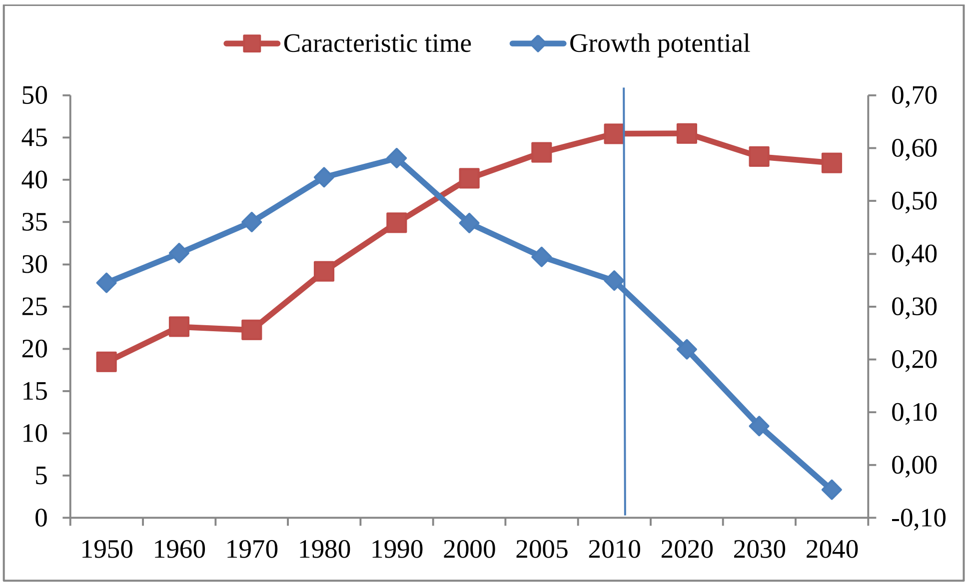

Figure 5: Growth potential (α) (right scale) and characteristic time (τ) (left scale): UN actual data (1950-2010) and UN projections (2010-2050). See Table 2.

If the characteristic time reflects a global tendency for women to have children at later ages (which might well be the case), by the year 2000, this age may be approaching already the age at which women become infertile. For the period 1950-2000 we can take an average value $\tau \approx 30.3$ years which would result in an effective $\tau^* = \tau / \cosh\alpha \approx 27.2$ years for the above period.

Finally Fig. 6 gives the actual UN data (and the 2004 projections) and the theoretical curves describing the time evolution of the world population (1950-…). The Malthusian projection for population growth beyond 1980 is also given for comparison: $P(t) = P(t_0)(2)^{(t-t_0)/25}$ with $P(1980) = 4.5\times10^9$ at $t_0 = 1980$.

The analysis presented in this work could suggest further research in the factors influencing birth rates in women and men, as well as death rates. Regarding birth rates, of course there is a maximum number of children a woman wish or is able to have. Also regarding death rates there is a maximum which human nature is made to live.

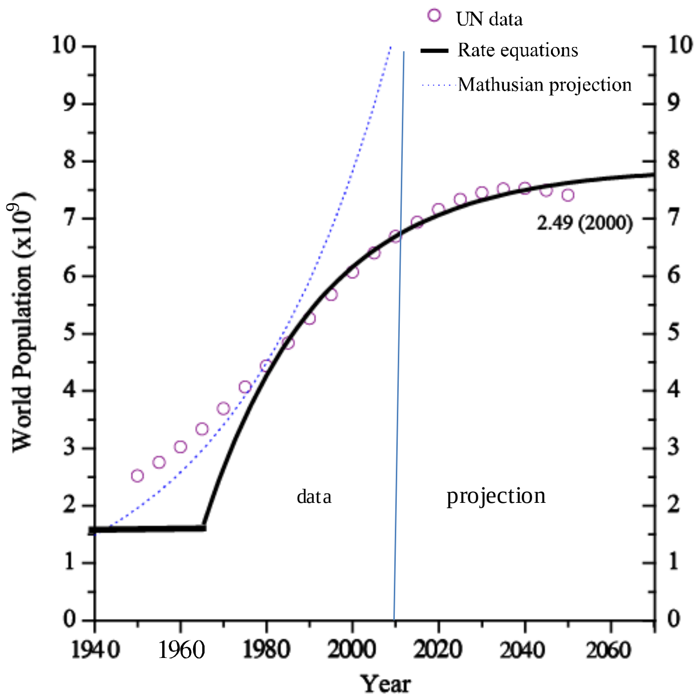

Figure 6: World Population $P(t)$ versus time. 1950-2010: UN data (circles); Rate equations (continuous curve) 2010-2050: UN projections (circles); Rate equations (long term projections) $(1.50 \le x \le 2.49)$.

5. Conclusion

The population increase in 1950-2010 should be attributed more to the transient decrease in death rate level (related to the increase in life expectancy) than to non existent increase in birth rate, which was decreasing consistently already even before the 1950's, even before chemical contraceptives and legalized abortion begun to play any role.

An exponential population growth can be discarded as totally unrealistic. Using fitting parameters extracted from recent UN population data, our rate equation solutions approach, which indicates that $x(t)=r_b(t)/r_d(t)$ may approach to one at $t \approx 2032 \pm 10$, suggests a decrease in world population beginning to take place about this time.

In 1960 population and world economic development was examined in *Science*. At about the same time, Foerster, Mora and Amiot (1960) reported in *Science* that that November 13th, 2026 would be the date at which world population would become infinity. The predicition was based upon an empitrical equation for the population with a denominator going to zero as time increases. Today, 2011, world population is approaching 7.0×10^9 and, according to the UN data it will be around 7.3×10^9 in the year 2026 and approaching a maximum somewhat later.[3] (Reports in Science on the UN Conferences on World Population at Bucharest and Mexico were given by Boersma (1975) and Lutz, O'Neill and Scherbov (2003).)

The evidence for 'negative momentum' in Europe's population around 2000 is a result of low fertility rates. Caldwell (2008) most recently, criticises Mathew Connelly's book 'Fatal Misconceptions. The Satruggle to Control World Population'. Eberstadt (1997; 2001), on the other hand, points out that the world today may confront an unfamiliar crisis: rapidly decreasing birthrates and declining life spans that might set back the progress of human developments. Work by other authors share this perspective (see for instance Chaunu 1997; Ulrich 2000; Yea 2004).

The UN Press Release reports world population figures for 2050 and 2100, *if* fertility in all countries converge to replacement levels. Total fertility for the world and for countries grouped by fertility level in 1965-1970, 2005-2010, 2045-2050 and 2095-2100 are given. It may be noted that a large decrease between 4.8 and 2.5 is recorded for 1965-70/2005-10, much larger than the (expected) low decrease between 2.2 and 2.0 for 2045-50/2095-2100.

We think a rate equation approach may provide a useful tool to simulate world population trends.

References

Asimov, I. (1972). *Asimov's Guide to Science*. New York, Basic Books.

Boersma, D. (1975). "World Population Conference in Perspective." *Science* **188**(4193): 1069-1069.

Caldwell, J. C. (2008). "Fatal misconception - The struggle to control world population." *Science* **321**(5892): 1043-1043.

Chaunu, P. (1997). "From explosion to implosion of the population - Vital peril." *Cahiers d'Economie et Sociologie Rurales* **181**(9): 1923-1933.

Duan, N., C. Arago, et al. (2010). "Ferroelectric Direct Energy Converters in Perspective." *Ferroelectrics* **400**(1): 321-335.

Eberstadt, N. (1997). "World population implosion?" *Public Interest* (129): 3-22.

Eberstadt, N. (2001). "The population implosion." *Foreign Policy* (123): 42-53.

[3] The lower most likely estimate: *The 2004 Revision*/ *The 2010 Revision*.

Foester, H.v., P.M. Mora, et al. (1960). “Doomsday: Friday, 13 November, A.D. 2026.” *Science* **132**: 1291.

Groombridge, B. and Jenkins M.D. (2002). *World Atlas of Biodiversity*. Prepared by the UNEP World Conservation Monitoring Centre. University of California Press, Berkeley, USA.

Lutz, W., B.C. O'Neill, et al. (2003). “Demographics: Europe's population at a turning point.” *Science* **299**(5615): 1991-1992.

Ulrich, R. E. (2000). “Explosion of the world population or implosion?” *Internationale Politik* **55**(12): 17-24.

Vian Ortuño, Á., Ed. (1994). *Introducción a la Química Industrial*. Barcelona, Editorial Reverte.

Yea, S. (2004). “Are we prepared for world population implosion?” *Futures* **36**(5): 583-601.